Fourier Analysis

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Introduction

- Welcome!
- Jean-Baptiste Joseph Fourier
- Importance and Applications in Various Fields

The Basics of Fourier Analysis

- Fourier Analysis is a mathematical technique used to decompose periodic and non-periodic functions into simpler components.
- Fourier Series: Representation of periodic functions using a combination of sine and cosine functions.
- Formula for Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right) \right)$$

- Fourier Transform: Extension of Fourier Series for non-periodic functions, converting functions from the time domain to the frequency domain.
- Formula for Fourier Transform:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Fourier Series

- Introduction to Fourier Series
- Explanation of periodic functions
- Representation of periodic functions using trigonometric series
- Formula for Fourier Series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right) \right)$$

Fourier Transform

Converting from the Time to the Frequency Domain

- Introduction to Fourier Transform
- Extension of Fourier Series for non-periodic functions
- Conversion of functions from the time domain to the frequency domain
- Formula for Fourier Transform:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

Frequency vs. Time Domain

- Time domain represents signals as a function of time, showing their evolution and characteristics.
- Frequency domain represents signals as a function of frequency, revealing different frequencies and their strengths.
- Advantages of frequency domain: identification of dominant frequencies and harmonic components, compact representation of signals, efficient data analysis.
- Illustration: In the frequency domain, complex signals show individual frequencies and amplitudes, such as a musical chord.

Signal Processing

- Fourier Analysis in signal processing has applications in filtering and isolating specific frequencies from a signal.
- Fourier Analysis is used in image and audio compression to reduce data size while maintaining quality.
- In image compression, high-frequency components are quantized or discarded based on their perceptual significance.
- In audio compression, less significant frequency components are selectively removed or quantized for reduced data size.

FFT - Fast Fourier Transform

- How does a computer use Fourier Analysis?
- Efficient algorithm for calculating the Fourier Transform
- Breaks down the calculation into smaller computations
- Exploits symmetry and periodicity properties for speed improvements
- Revolutionized digital signal processing for real-time applications
- Time complexity of O(n log n) enables processing of large datasets in real-time

Key Takeaways

- Fourier Analysis is a mathematical technique used to decompose complex signals into simpler components.
- Fourier Series represents periodic functions as a sum of sine and cosine functions.
- Fourier Transform extends Fourier Series to analyze non-periodic functions, converting them from the time domain to the frequency domain.
- Fourier Analysis has applications in various fields, including signal processing, image and audio compression, and data analysis.
- The Fast Fourier Transform (FFT) is an efficient algorithm for calculating the Fourier Transform, enabling real-time processing and analysis.
- Fourier Analysis finds applications in mathematics, including harmonic analysis and solving partial differential equations.

References

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