Circumscribing shapes with Jordan curve

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Euler Circle

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Simple Closed curve

Simple Closed curve is basically a curve that you can draw on a paper without lifting your pencil and begins and ends at the same point.

Definition

A simple closed curve is f:[0,1] $\rightarrow R^n$, where f is one-to-one and f(0)=f(1).

Simple closed curve is often referred as Jordan curve.

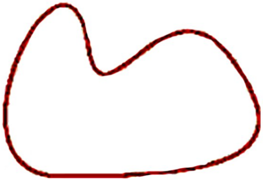


Figure: Example of a Jordan curve

"Nice Enough"

Definition

A Jordan curve J is "Nice enough" if for each point p $\in$ J, $\exists$ a coordinate system in which a piece of J containing p has function relation

All the Jordan curves that are in this presentation and that we study in the paper are "nice enough". Not "nice enough" are curves that are extremely fractal like. The extreme example of Jordan curve like one below would be not "nice enough".

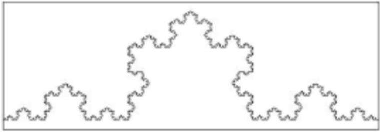


Figure: Example of what not "nice enough" looks similar to

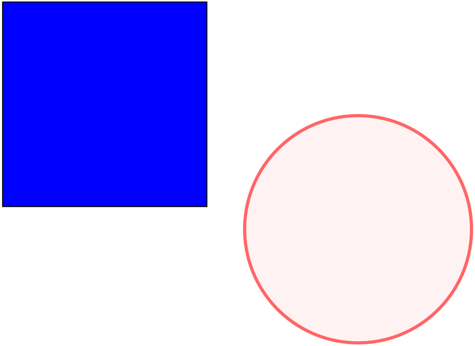
Jordan curve symmetric about origin

Theorem

A Jordan curve J symmetric about origin circumscribes at least one square.

Circumscribing another figure means that a figure contains the vertexes of points of another figure while maintaining congruence.

Square and Circle as examples



These shapes all circumscribe squares. In fact, square and circle inscribe infinitely many squares! What common characteristic do they have in common?

Extension of symmetry about origin

Let's generalize square and circle shown in previous slides to a Jordan curve in general.

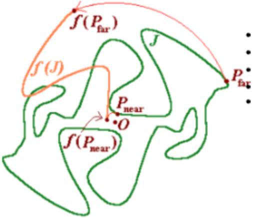


Figure: Shortened step of proof of Jordan curve with origin symmetry

Jordan curve in R^3

We've been dealing with Jordan curve in R^2. How about Jordan curve in R^3? Well.

Theorem

Jordan curve in R^3 that is differentiable at at least one point circumscribes many triangles similar to an arbitrary triangle.

Visualization of J in R^3

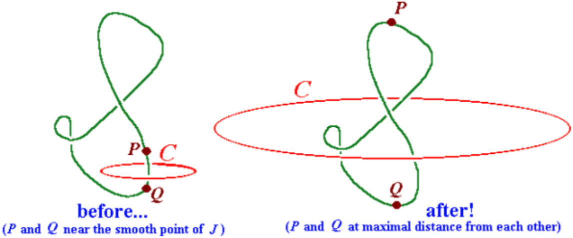


Figure: Jordan curve in 3D near differentiable point

Inscribed rectangle theorem

Theorem

A Jordan curve circumscribes at least one rectangle.

An essential function to this proof

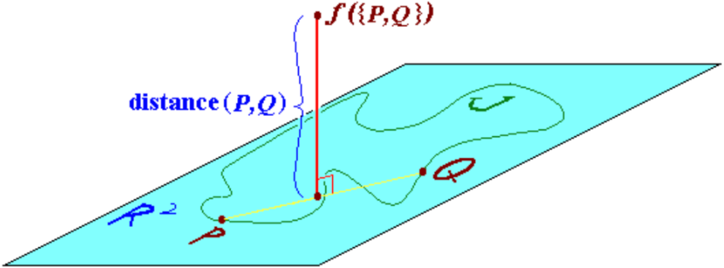


Figure: Mapping J to R^3

Square to Mobius strip to inscribed rectangle

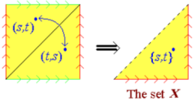


Figure 14. Folding across the diagonal

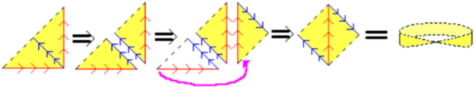


Figure 15. Making Mobius strip by gluing and twisting the square

Figure: Square to Mobius Strip

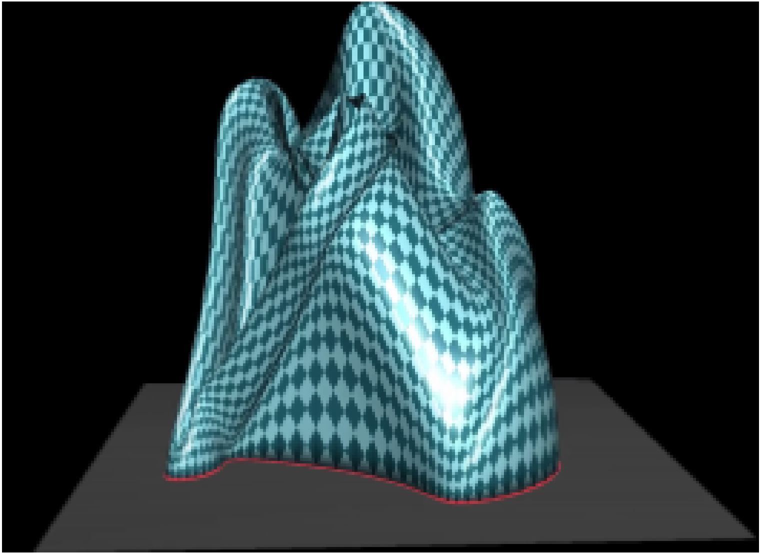


Figure: Mapping Mobius strip to R^3

Recent progress

The paper describes some of the proofs related to the recent progress regarding convex Jordan curve's ability to contain a cyclic quadrilateral.

Special Thanks

I want to thank Simon, Rajiv, and everyone that was supportive when I asked for help in Euler circle. People's enthusiasm for math in this class helped me understand all the concepts very well. I will show appreciation by ending the presentation with a duck.