Origami Constructible Numbers

Medha Ravi

Euler Circle Independent Research

2022

Straightedge and Compass Constructions

1. Creating the line through two existing points
2. Creating the circle through one point with center another point
3. Creating the point which is the intersection of two existing, non-parallel lines
4. Creating the one or two points in the intersection of a line and a circle (if they intersect)
5. Creating the one or two points in the intersection of two circles (if they intersect).

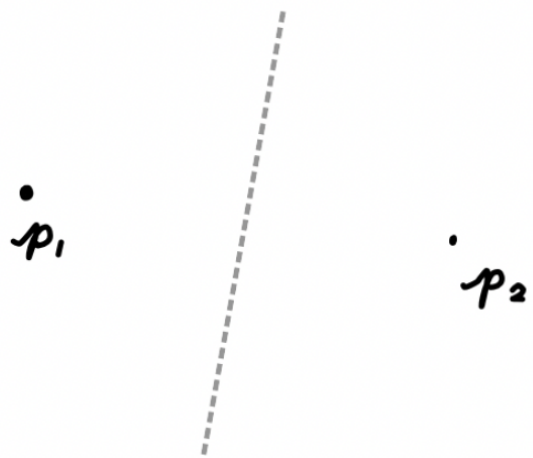
What are Origami Constructible Numbers?

A number x is origami constructible if, given two points, 0 and 1 on the $x-$axis, a series of folds can be performed in which two creases intersect on the paper such that the distance between the two creases equals x.

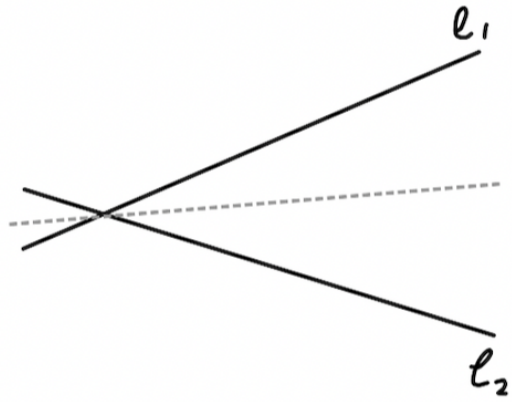
Origami's Expansion of the Constructible Field

Given two distinct points p1 and p2, there is a unique fold that passes through both of them.

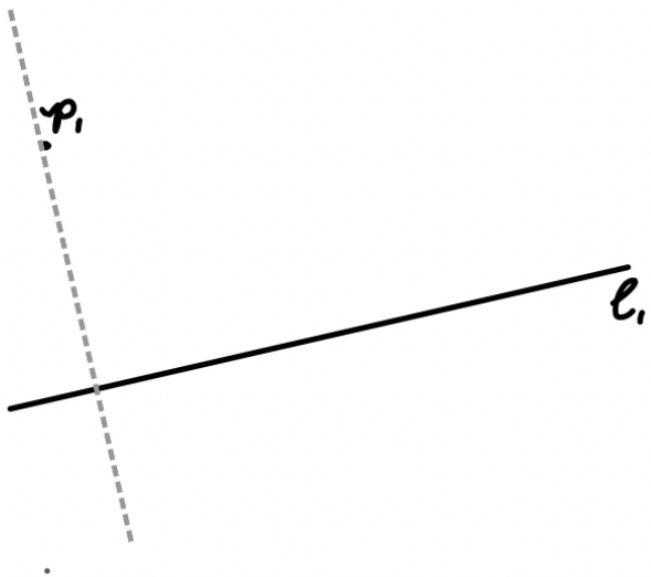
Given two distinct points p1 and p2, there is a unique fold that places p1 onto p2.



Given two lines l1 and l2, there is a fold that places l1 onto l2.

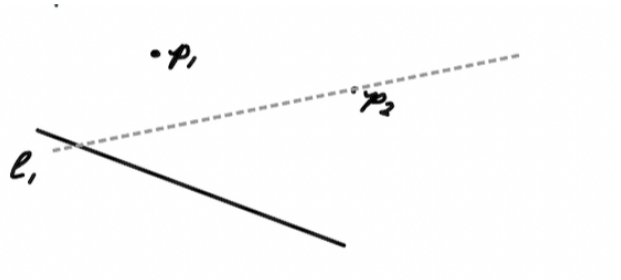


Given a point p1 and a line l1, there is a unique fold perpendicular to l1 that passes through point p1.

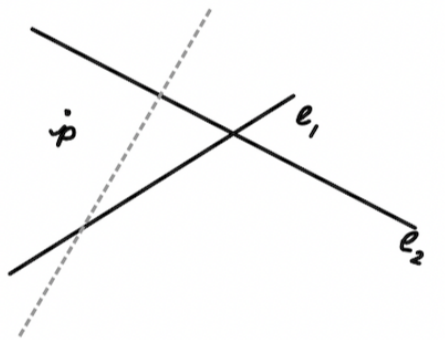


Origami's Expansion Pt 2

Given two points p1 and p2 and a line l1, there is a fold that places p1 onto l1 and passes through p2.

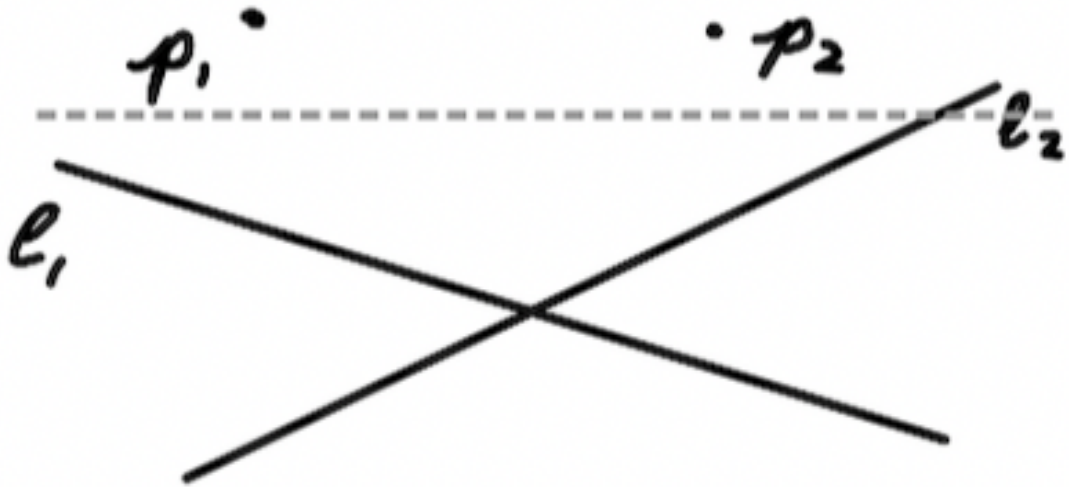


Given one point p and two lines l1 and l2, there is a fold that places p onto l1 and is perpendicular to l2.

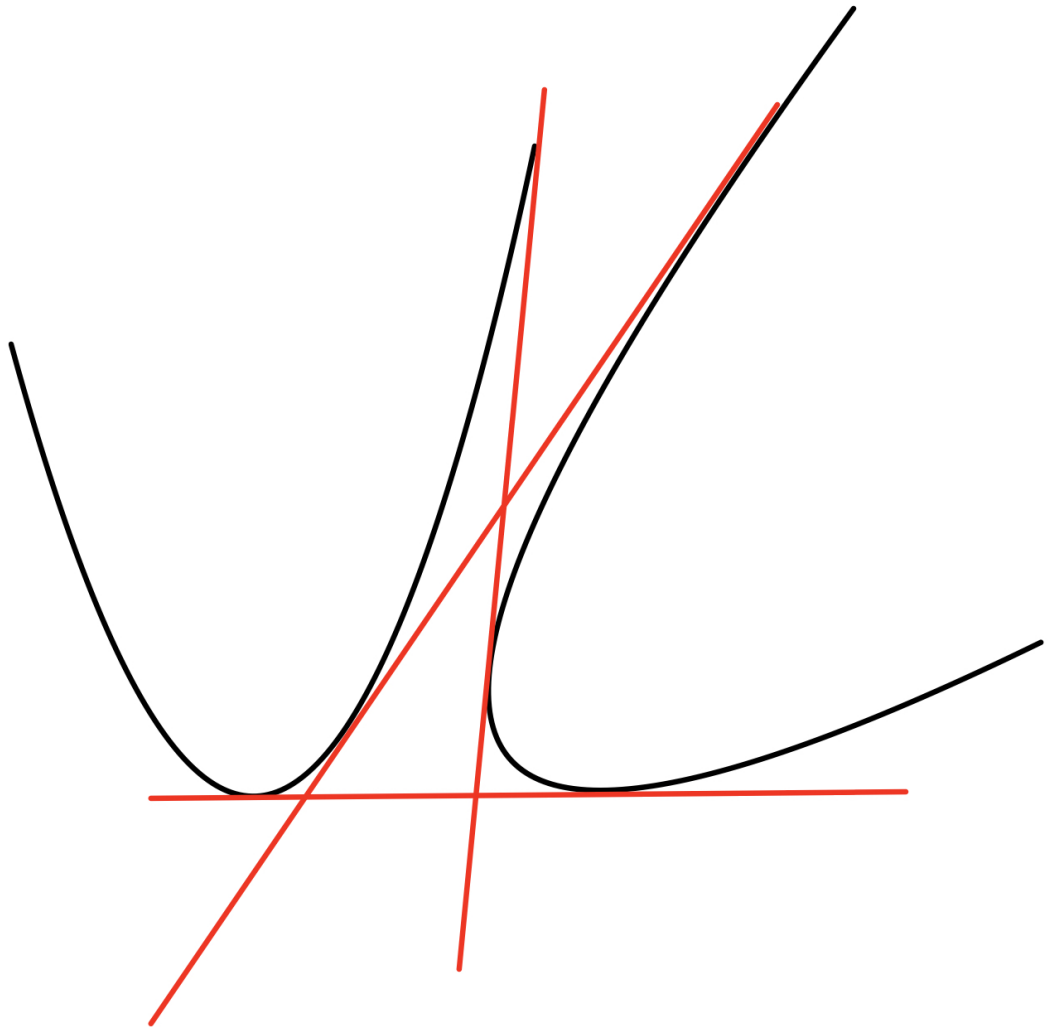


Beloch Fold

Given two points p1 and p2 and two lines l1 and l2, there is a fold that places p1 onto l1 and p2 onto l2.



Cubics and the Beloch Fold



Interesting Constructions

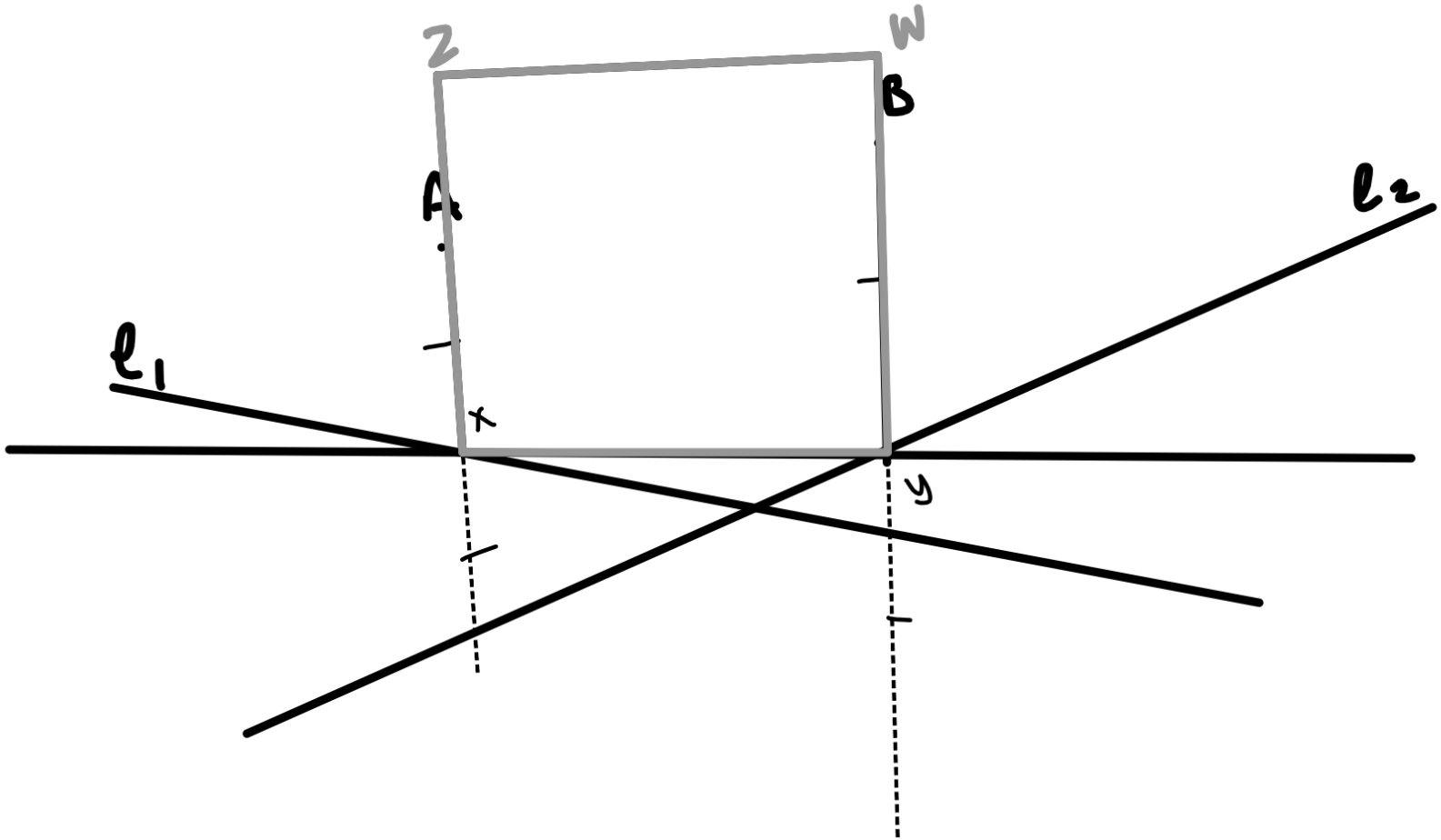
Doubling the cube, $\sqrt[3]{2}$, and the Beloch Square

Doubling the cube is one of the three delian problems:
Given a cube with volume v, construct a cube with volume $2v$.
To do so, we need to construct the $\sqrt[3]{2}$.

The Beloch Square

Given two points A and B and two lines l_1 and l_2, a Beloch square is a square XZWY such that X and Y lie on l_1 and l_2 respectively, A lies on line XZ and B lies on line YW.

Doubling the cube, $\sqrt[3]{2}$, and the Beloch Square

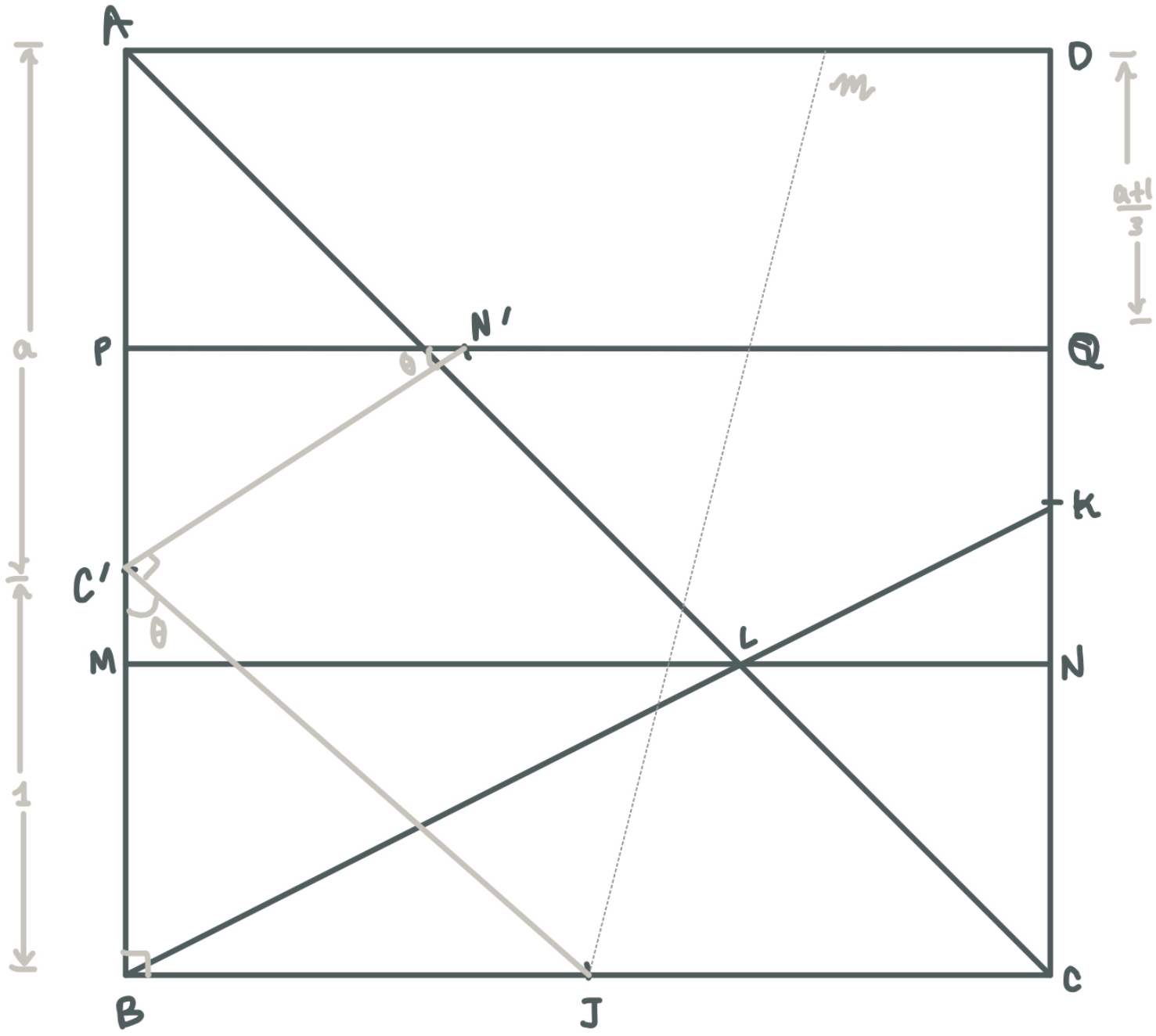


$\sqrt[3]{2}$

For a square $ABCD$,

1. Construct the midpoint J of side BC.
2. Construct midpoint K of side CD
3. Find the intersection L of lines AC and BK
4. Construct a line MN parallel to line BC through L
5. Construct a line PQ parallel to line MN halfway between MN and AD.
   We have now divided side AB and DC into thirds with points M and P and N and Q, respectively.
6. Use axiom 6 to create fold m, placing C on AB at C' and N on PQ at N'
7. AC' is $\sqrt[3]{2} \cdot BC'$

$\sqrt[3]{2}$



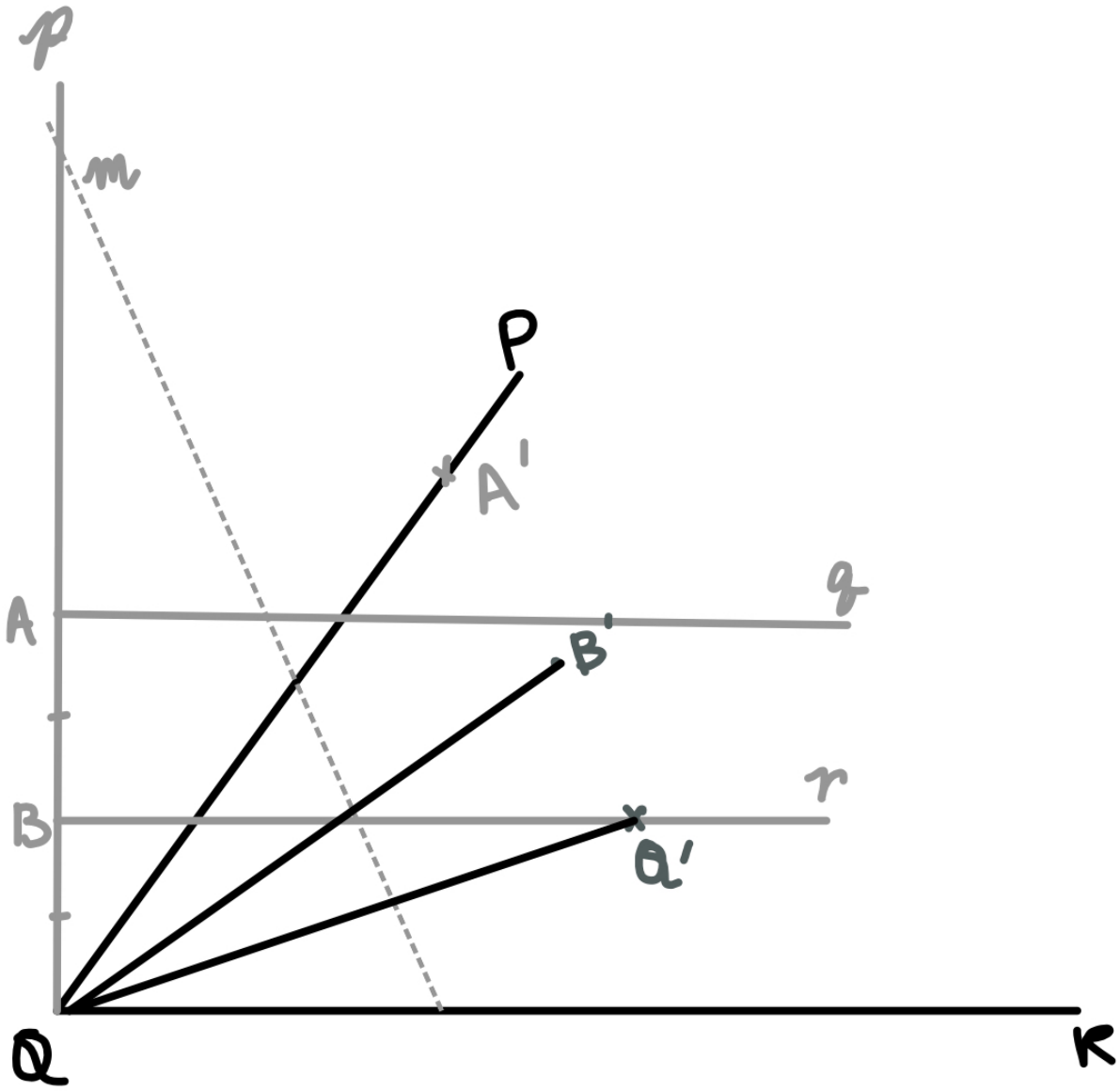
Angle Trisection

Given angle $\angle PQR$:

1. Allow line p to be the perpendicular to QR at point Q.
2. Let the foot of any perpendicular q to p be A.
3. Let the foot of a perpendicular r to p, B be a point equidistant from A and Q.
4. We construct fold m placing A onto PQ at A' and Q onto line r at Q'
5. Let point B' be the image of B reflected across fold m.

PQB', $B'QQ'$, and $Q'QR$ equally trisect angle PQR.

Angle Trisection



Squaring the Circle

Extra Articles

- http://origametry.net/papers/amer.math.monthly.118.04.307-hull.pdf
- https://www.math.miami.edu/ armstrong/461sp11/ImpossibleConstructions.pdf
- https://www.cs.mcgill.ca/ jking/papers/origami.pdf