

Euler didn't mind to prove lemma due to the fact that Euclid had already.

Lemma 1 This lemma is already proved by Fermat, so Euler doesn't feel like proving it. This lemma is that the product of distinct primed can never be a power of any kind (for example a square).

Lemma 2 This lemma states that if $a^2 + b^2$ sums up to a square, such that a and b are coprime. Then $a = p^2 - q^2$, and $b = 2pq$, such that p, and q are coprime. When p, or q are even, and the other is odd.

Proof of Lemma 2

Because $a^2 + b^2$ is a square,

set its root equal to $a + bq/p$

where I have that q/p

expressed in the smallest terms such that p and q are related to each other prime numbers are. After the equation has been established,

$a^2 + b^2 = a^2 + 2abq/p + b^2 * q^2/p^2$

be pb. Therefore, will

a: b = (pp - qq): 2pq.

The numbers pp - qq and 2pq are either prime or have the one another common divisor 2. In that case, in which pp - qq and 2pq to each other prime numbers are what happens when one of the numbers p and q are straight was, the other odd, it is necessary that

a = pp - qq and b = 2pq

is is because a and b are fixed as mutually prime numbers. In that case

but in which the numbers pp - qq and 2pq have the common divisor 2,

what will happen if each of the two numbers p and q was odd (Both effects can not be equal, because they are assumed to be among the first), a =

pp-qq

2

and b = pq. But put $p + q = 2r$ and

$p - q = 2s$; r and s will be prime numbers and one will be prime numbers

he is straight, the other odd, where from

$$a = 2rs \text{ and } b = r^2 - s^2$$

which expression, because it matches the first one, indicates when $a^2 + b^2$ was a square and a and b are prime numbers that the one of which is the difference between two prime squares to each other, of which one is even, the other number, but equal to the double Product from the roots of these squares is. It means that $a = p^2 - q^2$ and $b = 2pq$ while p and q are prime numbers to each other, one of which is even, the other is odd.

Q.E.D.

COROLLARY 1

So if the sum of two mutually primitive squares was a square, it is necessary that the one square is straight, the other but odd; it follows that the sum of two odd squares is not a square can.

COROLLARY 2

So if $a^2 + b^2$ is a square, one of the numbers a and b , z. B. a , odd, but the other b is straight. But odd becomes $a = p^2 - q^2$ his and straight $b = 2pq$.