A Q-Analog of a function f(x) is a function f(q, x) which equals f(x) when q = 1.

The prefered Q-Analog of n is (1-q^n)/(1-q), which can be shown to be a Q-Analog of n both by L'Hospital's Rule, and by geometric series. The notation of the Q-Analog is $[n]_q$. To show that it works, use the L'Hospital's Rule to differentiate the top and bottom, getting nq^(n-1), and because q is 1, therefore this becomes n. To use the geometric series, turn it into 1/(1-q)-(q^n)/(1-q), and then expand using the fact that 1/(1-q) = 1+q+q^2+q^3..., getting 1+q+q^3+...q^(n-1).

The Q-Fibonacci numbers are a Q-Analog of the Fibonacci numbers, which use the formula $F_{n+1}(q)=F_n(q)+q^{(n-1+a)}F_{n-1}(q)$. To see how this is made, we can make a board of length n, and tile it with squares and dominoes. Those are the fibonacci numbers. Then, we can define the weight of a board to be $w(x) = \sum_{All\ dominoes\ at\ (i,i+1)} i$. Then, the Q-analog of fibonacci is

$$f_n(x) = \sum_{All\ Tilings\ t\ of\ n} q^{w(x)}$$

For example,
|=square
[]=domino
|||[] has a weight of 4.

To start getting identities that work with the Q-Fibonacci numbers, we must define the Q-Fibonacci shifted numbers, which have the formula $F^{(a)}_{n+1}(q)=F^{(a)}_n(q)+q^{(n-1+a)}F^{(a)}_{n-1}(q)$. These represent the amount of ways for a board which is shifted by a number a. You can multiply this by another board to get a board to get a board of size m+n with no domino at place m or n.

Here is an example of a identity.
$f_{2n+2}=f_n^2+f_{n+1}^2$.
This is obvious because a board could either have a domino in the middle or not. The q-analog version is $f_{2n+3}(q)=q^{n+1}f_{n+1}(q)f_{n+1}^{(n+2)}(q)+f_{n+2}(q)f_{n+2}^{(n+1)}(q)$.

On the left hand side, there is the number of weighted ways to make a board of size 2n+2. On the right hand side, we can consider position n+1 on the board. If there is a domino there, it contributes n+1 to the weight, and we have $f_{n+1}(q)$ weighted tilings to the left and right, but the ones on the right have to be shifted by n+2. If that domino is not there, on the other hand, then there just is $f_{n+2}(q)$ weighted tilings, and then we just shift the ones on the right side over by n+1.

There are also the Lucas numbers, which are the number of ways of tiling a bracelet of size n with squares and dominoes. We can say that it is “in phase” if there is no domino in the nth position, and we can say that it is “out of phase” if there is one. This satisfies the same recurrence as the fibonacci numbers, but has different starting conditions, with $L_0=2$, and $L_1=1$.

This can be shown, because L_0 could either be in phase or out of phase, giving 2, while L_1 can only have one tiling, a square. Then, the recurrence can be shown because if there is a domino at the n position, remove it, and therefore it is now a n-2 board, but if there is a square, do the same and have a n-1 board. We can do the same weighting method as the q-fibonacci numbers, and have the same recurrence.

Then, there is quantum calculus with the Q-Derivative and H-Derivative. These derivatives both do not have limits, and the Q-Derivative can not be evaluated at zero. The Q-Derivative is $\frac{f(qx)-f(x)}{qx-x}$. This derivative is very useful when finding the derivative of x^n. Just plug in, getting $[n]_q x^{n-1}$, and because $[n]_q$ is the q-analog of n, this becomes nx^{n-1} when q goes to 1.

You can invent a Q-Analog of almost anything, although Q-Analogs usually don't have uses, only to package information in a better way.