What is a differential poset?

A poset is differential, and if you want to be more specific r-differential if it satisfies the below conditions.

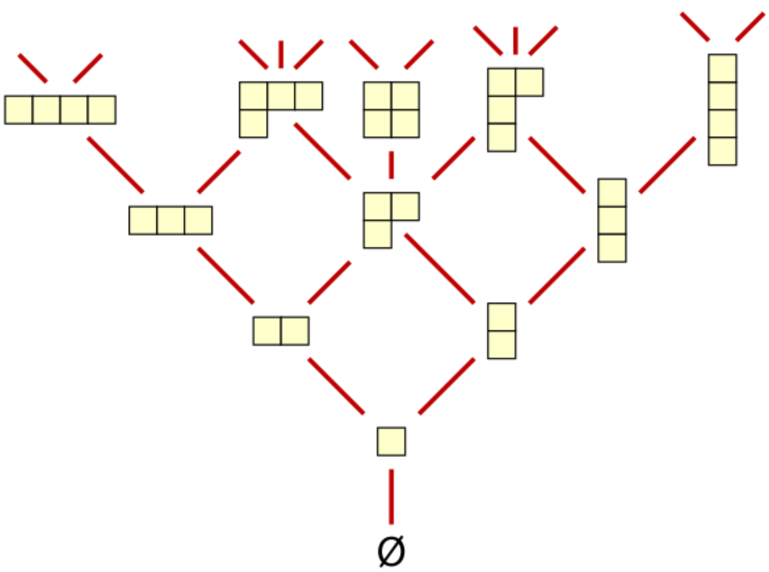
1. P is graded and locally finite with a unique minimal element
2. For every 2 elements $x, y \in P$, the number of elements covering x, y is equal to the number of elements covered by x, y
3. for every element x of P, the number of elements covering x is exactly r more than the number of elements covered by x.

<u>Definitions</u>

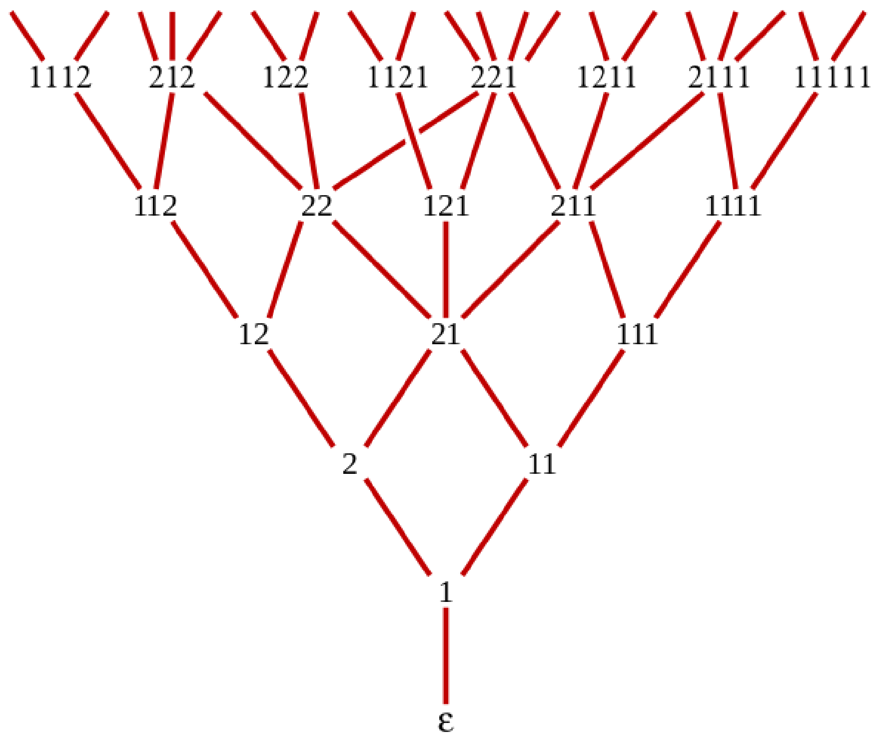
a. Graded poset - has a rank function
b. Locally finite poset P such that for all $x, y \in P$, the interval $[x, y]$ consists of finitely many elements.

<u>Young's Lattice</u>

The poset of the number of partitions ordered by inclusion



Young-Fibonacci Lattice



The Young-Fibonacci Lattice shows the partitions of an integer into 1 and 2s.

These examples of the posets are 1-differential

Unsolved Problem

Are there any differential lattices that are not products of the Young's lattice and the Young Fibonacci lattice.

Properties

a. The number of paths that have length of $2n$ in the Hasse Diagram of a poset P beginning and ending at the minimal element equal to $(2n-1)!!$ (this is intended to be a double factorial). Note this formula works for 1-differential posets. For r-differential posets, the count is $(2n-1)!! \cdot r^n$
b. In the Hasse diagram of P, the number of paths with length of $2n$ that start at the minimal element and such that the first n steps are covering relations from a smaller to larger element of P is $n!$. This assumes that the poset is 1-differential.

        i. If we were to extend this to r-differential posets, it would be $r^n * n!$, which is pretty intuitive

c. The number of upward paths of length n in the Hasse diagram of a poset P that begin with the minimal element is equal to the involutions in the symmetric group of n letters.
    i. With an r-differential poset, the exponential generating function is $e^{rx+\frac{x^2}{2}}$.

<u>More Definitions</u>

1. Involution - A function f that has its own inverse. Basically, after 2 moves, it cycles back. In function terms, that is equal to $f(f(x))$