# mnk Games

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## 1 Intro

I define the class of *mnk* games to be games played on an $m$ by $n$ grid; players take turns placing their counters on the board squares and win once they put $k$ of their own counters in a row.

In this writeup, I discuss certain examples of the *mnk* games as well as their winning strategies, and then move on to the algorithms people used to prove the existence of these strategies.

## 2 *mnk* games

The most famous case of mnk games is the 3-3-3 game, more commonly referred to as tic-tac-toe. Many people have played this game, and if both players play optimally the game will always end in a tie. In this case, brute-forcing through the various combinations shows that players will find themselves in a tie. In the 4-4-3 game, however, the first player always; by moving twice in the four center squares, they will always have an open slot to win.

### 2.1 Gomoku

The adult version of this is called GoMoku, known in *mnk* terms as the 19-19-5 game. In this case, it is much harder to brute-force through all the possible combinations of counters, but playing this game a few times reveals some basic strategies that must be adhered to. In doing so, it becomes necessary to introduce some terminology.

For any *mnk* game, call a (*k-2*) in a row a *POT* and a (*k-1*) in a row a *SHOT*. If a *POT* or *SHOT* has opposing counters on both ends, preventing it from becoming any longer, that line is **BLOCKED**. If it has opposing counters on one end, call it **CLOSED**; if none, call it **OPEN**. I will refer to the two players as A and B.

It is apparent that, if A has an open *SHOT*, then B can do nothing to prevent A from winning unless he has his own *SHOT*. Similarly, if A creates two open *POTS* in one move, B can do nothing to prevent one from becoming an open

*SHOT*. B must respond to A's closed *SHOT* or open *POT* whenever possible; otherwise A advances those to either the win or an open *SHOT*.

### 2.2 Focusing on k

Given that we increased *k* from 3 to 5, one might wonder if there exists a winning strategy for all k, presuming a sufficiently high m and n. While not known for 6 or 7, it has been shown that k ¿ 7 always results in a tie. The argument used to deduce so is by a pairing strategy, which focuses on pairing all the squares together in such a way such that if A moves in a pair, B moves in that pair as well. I hypothesize that k=6 and k=7 are first-player wins on an infinite board, noting that there exist 8 cardinal directions that the line can take; since 6 and 7 are less than 8, the first player can always find a line through which they can create a board. Note that it would be sufficient to show this condition on a board of finite and small m and n.

### 2.3 Multidimensional mnk games

We briefly discuss the implications of extending this category of game to the third dimension or even higher. Firstly, we denote these games in the (n+1)-tuple (m1, m2, ..., mn, k), where n is the number of dimensions and m1...mk ¿ 1 are the lengths of the board in each direction. Note that the game (3, 3, 3) is equivalent to the game (3, 3, 1, 3); this is equivalent to changing the shape of the squares, without inherently affecting the game itself. The most common version of multidimensional games is the (4, 4, 4, 4) game; this has also been shown to be a first-player win. In addition, for any k, there exists a sufficiently large n such that the game (k, k, ..., k, k) also results in a first-player win.

## 3 Computer Algorithms

Lastly, we discuss the various algorithms used to discover these strategies, as implemented by others. Brute force is simply going through all possible configurations and seeing their results, then working from the start position all the way down to the end; knowledge-based search takes various endgame positions and works backwards to enter the start position; Threat-space search takes various positions and discovers if someone can win by repeatedly creating threats that the other person eventually cannot respond to. Proof-number search assigns game configurations a Proof and DisProof number, and turns the game tree into a binary AND/OR tree consisting of these numbers; by choosing the right nodes representing the optimal moves, the search is made more efficient.

## 4 Parting Thoughts

While mnk games have been analyzed before, there still exists much we do not know about them. In particular, there exists no strategy for Gomoku that humans can replicate, nor has there been much research done on multidimensional mnk games apart from mentioned above. Despite this, I learned a lot about mnk games these past few weeks and plan on experimenting with the algorithms to try and create a conclusion on the 6-in-a-row eventually.

## 5 Sources


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