BIDDING GAMES

KARTHIK BALAKRISHNAN

1. Introduction

Bidding Games are combinatorial games in which both players have bidding chips to decide who gets to make a move. Rather than players alternating players can decide how important it is for them to move and bid accordingly. In bidding games, there are two main topics, discrete and non-discrete games. For this presentation let us look at discrete games, where each player has a whole number of chips. Players will simultaneously bid some number of chips and the player who bids more will give his or her bid to the other player and make a move. We can represent any bidding game G as $G(a, b)$ where a is the number of chips the first player has and b is the number of chips the second player has. One problem that we already see is what to do in the case of a tie. To rid the game of uncertainty we use the *tie-breaking advantage.*

Definition 1.1. The *tie-breaking advantage* is a piece that allows the player using it to decide who wins a tied bid. If the player uses the advantage, they must give the other player the advantage. If they allow the other player to move, they get to keep the advantage.

We can represent the advantage in the game notation, using a star. In game G, if the first player has the advantage we can represent it as $G(a^*, b)$. For the sake of simplicity let us call the first player Alice and the second player Bob.

Theorem 1.2. *If Alice wins in $G(a, b^*)$ she must also win in $G(a^*, b)$.*

Proof. In the latter game, Alice plays as if she did not have the advantage until the first tie. When the first tie occurs Alice declares herself the winner, and that results in the game $G(a, b^*)$. Therefore, Alice has a winning strategy by assumption.
One might ask the question,"What is more valuable, the tie-breaking advantage or an extra bidding chip?". To test this, let us compare a chip against the tie breaking advantage.

Theorem 1.3. *If Alice wins in $G(a^*, b + 1)$ she must also win in $G(a + 1, b^*)$.*

Proof. Alice's strategy in the latter game, is to play as she would in the former game, except when the time comes for her to use k chips and the tie-breaking advantage, she will bid $k + 1$ chips, and thereby win. Thus, an extra bidding chip is more valuable than the advantage.